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Volume 6

A Dialogical Framework for Analogy in Legal Reasoning

The Ratio Legis and Precedent Case Models

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Law and Society Series Editors
Robert L. Heiner rheiner@plymouth.edu
John Woods john.woods@ubc.ca

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The Ratio Legis and Precedent Case Models

Hans Christian Nordtveit Kvernenes

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Contents

Introduction

Background

The present project is twofold. First, it gives a thorough representation of the concept of analogical reasoning in law by introducing and comparing contemporary theories regarding the subject. Second, it provides an independent analysis of analogical reasoning in the framework of immanent reasoning.

The first part provides explanations and comparison of six contemporary theories of analogical reasoning in law. These theories are categorised into schema-based theories and inference-based theories. Schema-based theories of analogical reasoning capture the notion of analogy by a description of a rule or schema. Inference-based theories on the other hand explain analogical reasoning as a distinct way of reasoning. Based on this distinction, the project compares the theories by how they handle the notions of horizontal and vertical relations and by how they analyse multiple competing analogies.

The second part of the project provides an independent analysis of analogies by utilising the framework of immanent reasoning. Immanent reasoning is described, together with an explanation of other notions that are relevant for the given analysis. The project then introduces two kinds of analogical reasoning, general precedent-based reasoning and precedent-based reasoning with heteronomous imperatives. These kinds are first analysed in the general formulation of constructive type theory (CTT) and is then given an alternative

formulation in its dialogical interpretation. Following this, we introduce a discussion on the advantages of utilising this framework for analysing legal reasoning in general and analogical reasoning in particular.

Based on a variant of the principle of proportionality, the present project provides a new analysis of analogical arguments in the framework of immanent reasoning. By utilising the formalisation of moral conditionals, we show how we can achieve an analysis of analogical reasoning. Because of the particular notion of dependent types in CTT, this approach also allows for formalising initial conditions and thereby an explicit notion of permitted analogy. The dialogical interpretation takes this one step further as this allows for representing this feature as an individual condition for the particular form of the introduced analogy.

The inclusion of initial conditions is a new feature not known to have been previously introduced in any other contemporary analyses of analogy and the particularity of this project is that it provides further meaning explanations of analogical reasoning that includes an initial permission in a simple and natural way, closely related to actual legal practice.

Contemporary theories of analogy

Problems of analysing analogical reasoning have received much attention in Antiquity and Middle Ages, notably by Aristotle's analysis of *proportionality*. And in recent years, these questions started again to receive attention. In the scientific context, this modern attention to questions about analogy can be traced back to Hesse's (1965) *Models and Analogies in Science*, where she provided a thorough analysis regarding the use of analogies in different scientific contexts. Somehow distinct from this tradition, another contemporary interest in such questions stemming from legal theory also emerged.

In the context of law, the use of analogies is widespread and theoretical problems regarding their analysis have the last 30 years received attention from researchers across multiple disciplines, such as philosophy, legal theory and computer science. Theorists have from different perspectives tried to analyse the notion of analogy and its use in legal argumentation. Some theorists attempt to unite theories about analogy coming from both the legal and the scientific contexts, while others try to apprehend it only as a legal phenomena. Theorists also differ in what they consider to be the object of their analyses. Some take the perspective of a particular legal framework, while others intend to have a more general scope.

Broadly, we might distinguish between theories that explain analogical arguments as regular deductive arguments with some particular premises and theories that explain these arguments as a distinct form of reasoning. We will call the first kind *schema-based* theories and the second kind *inference-based* theories. In the schema-based theories the goal is to identify a *schema* or *rule* that enables us to consider the analogical argument as a valid deduction. In the inference-based theories, the goal is to identify analogical reasoning as a particular form of *reasoning* or *inference*.

Brewer (1996), Alchourrón (1991) and Woods (2015) describe three important schema-based theories. In «Exemplary Reasoning: Semantics, Pragmatics, and the Rational Force of Legal Argument by Analogy», Brewer provides a theory of analogical reasoning that became widely influential in the debate about the use of analogies in contemporary Common Law, particularly in the Anglo-American tradition. Even earlier, Alchourrón wrote a paper called «Los Argumentos Juridicos a fortiori y a pari» that provided a logical analysis of two kinds of analogical arguments. This paper became very influential in the Spanish-speaking (and to some extent German-speaking) academical debate, though since the paper never has been translated to English, its influence in the English-speaking context

has unfortunately been marginal. The book *Is Legal Reasoning Irrational?: An Introduction to the Epistemology of Law* by Woods is a very recent work that attempts to analyse the legal understanding of analogy in light of insights stemming from the theory of science.

Bartha (2010), Prakken and Sartor (1996) and Rahman and Iqbal (2018) provide three important inference-based theories. In *By Parallel Reasoning: The Construction and Evaluation of Analogical Arguments*, Bartha provides a formal model for analogical reasoning in a wide range of areas, from mathematics to everyday reasoning, including legal arguments. It has recently shown to be a very influential general approach to questions of analogy. Prakken and Sartor develop in multiple papers a particular dialogical theory that comes from a computer scientific perspective on legal argumentation generally. Rahman and Iqbal provide in the paper «Unfolding Parallel Reasoning in Islamic Jurisprudence (I) epistemic and dialectical meaning in Abu Ishaq al-Shirazi's system of co-relational inferences of the occasioning factor» a contemporary logical analysis of analogical reasoning stemming from the often neglected, but rich tradition of Islamic legal theory.

The different theories do indeed share many aspects, though there are also important differences, both between the schema-based theories and the inference-based theories, and across the individual theories. Some differences can be explained by the point that some theories reduce the notion of analogy to a schema, while others considers them as a particular kind of inference. A noticeable point is however how the different theories deal with multiple, competing analogies. In this aspect, the theories varies greatly. Some reject that this should be a part of the formal framework. Some introduce a particular formal concept to create a systemised hierarchy, while some consider that the competing analogies should be decided by the use of an analogical argument in itself.

The framework of Immanent reasoning

Constructive type theory was developed by Martin-Löf (1984) in order to have a language to reason constructively both about and with mathematics. The idea is to have a system where you do not distinguish between syntax and semantics in the same way as it is traditionally done. This enables us to keep meaning and form on the same level and therefore also interact with each other in a way that is explicit in the language itself.

Dialogical logic should not be considered a logical system by itself, but rather a framework where we can interpret different logical systems. It may be considered as a general approach to meaning, and we can therefore use it to develop and compare different logical systems. The idea is to consider meaning as being constituted in the argumentative interaction between two agents. We may trace this idea back to later Wittgenstein and his notion of *meaning as use*. The interpretation of constructive type theory in the dialogical approach is what is called 'immanent reasoning', where its most recent version has been developed by Rahman, McConaughey, et al. (2018).

The connection between CTT and dialogical logic seems to be strong. Again, we can refer to Wittgenstein and his claim that we should not position ourselves outside language when trying to determine meaning. In CTT we do not consider syntax and semantics to be distinguished in a similar way as in classical logic. For propositions, we have inference rules and not distinct syntactical rules that describe when a proposition can be formed. In the dialogical approach, the meaning is determined by how it is used in interaction. Because of this, we may consider dialogical logic to be a pragmatical approach to meaning and semantics. If we link CTT to the dialogical approach, we do only consider syntax to be a kind of semantics, but that both actually belong to the pragmatics. What we end up with is a system where we do not distinguish syntax, semantics and pragmatics as separate domains. Syntax, semantics and pragmatics are all essentially related to interaction and their

meaning should be understood in terms of a normative obligation of interaction between the players, which in turn is a moral notion. In this sense, logic is considered to be a result of the moral obligation of interacting. It is not something essentially fundamental, but rather found in the investigation of ethics.

The dialogical approach describes a dialogue between two players. These players may be called the 'Opponent' and the 'Proponent'. The two players argue on a thesis. A thesis is a statement that is subject for the dialogue. The Proponent begins by stating the thesis and the Opponent will try to challenge it. The Proponent will again try to defend this thesis from the Opponents attacks. The notions of challenging and defending is then used to establish the dialogical understanding of meaning.

Structure for formalising analogy in immanent reasoning
In immanent reasoning there is a distinction between formation rules and particle rules. In order to represent analogies in this framework, we will use this distinction and consider the formation rules to represent initial conditions that might be imposed on the analogy and particle rules to represent a principle coming from Aristotle's notion of proportionality. This proposal of the implementation of analogies in immanent reasoning will be an essential part of this project.

In this part we will first analyse analogical reasoning by providing two different kinds of conditions that need to be satisfied. The first kind of condition describes a permission for the utilisation of analogical argumentation in this precise case. In many situations the use of analogies in the legal argumentation process is restricted. This can be for several reasons, like constitutional limitations or because other legal rules already answer this particular legal question. This restriction is not found in other analyses of analogical argumentation and we consider this to be a newly introduced feature, introduced in this project. The second kind of condition is one that we find in most contemporary works regarding the logical foundation of analogical

reasoning. This is what will be called the efficiency requirement or the proportionality-principle. It will be implemented as ways for the challenger to attack the thesis of the defender by providing a counterexample.

The methodology used for analysing analogical argumentation in immanent reasoning is to describe general aspects relevant for arguments by analogy. This comes partially from the comparison of theories in the first part, but we will also argue for including the new aspect called *permission of the analogy*. This aspect is present in the literature from a legal point of view, but absent in the contemporary logical analyses of analogical reasoning. It is the CTT approach to consider both meaning and form in the same language that enables us to also include this in the analysis in a simple way.

Structure of book

The book is structured in three main parts. The first part gives a general overview over the notion of analogy and explains different contemporary theories proposed for analysing analogical reasoning. The second part gives an overview over immanent reasoning, which is the logical framework that is used as a basis for the included analysis. Based on the two preceding parts, the third part presents this proposal for formalising analogical reasoning.

Presentation of present theories
The first part of the project describes and compares different theories of analogical reasoning in respect to how they handle different kinds of analogies. It starts by a very brief historical introduction related to the concept of analogy and then goes on by describing some relevant terminological distinctions based on contemporary legal theory that will provide useful for the rest of the work. The different theories of analogy in the second chapter will be categorised in schema-based theories and inference-based theories. The described schema-based theories are the ones by Brewer (1996), Alchourrón (1991) and

Woods (2015). And the described inference-based theories are the ones by Bartha (2010), Prakken and Sartor (1996) and Rahman and Iqbal (2018). The part then finishes with a comparison in respect to how they represent what will be called horizontal and vertical relations and how they handle multiple analogies.

Theoretical background
The second part provides a thorough description of the theoretical framework of immanent reasoning, a very recent interpretation of the constructive type-theoretical framework by Martin-Löf (1984). CTT enables us to describe the interaction between form and meaning in a way that standard logical frameworks are not able to express. The idea was to develop a language where we can reason constructively in the same time with and about mathematics. CTT is then a powerful language that enables the formulation of hypothetical judgments that are not only dependent on objects, but also on categories by its notion of dependent types. Immanent reasoning is presented here in its last version, as given in Rahman, McConaughey, et al. (2018). This framework gives a sophisticated interpretation of both formal and informal reasoning by the means of a dialogical conception of truth. The idea of the dialogical approach is to consider meaning not relevant to some abstract model, but as argumentative moves in a play. By combining this dialogical approach with CTT we get a framework that is able to express the powerful notion of hypothetical judgments from CTT in a comprehensible way, that corresponds well together with the actual utilisation of argumentative moves.

A dialogical interpretation of analogy
The last part of the book describes the implementation of analogical reasoning as presented in the first part, by the framework of immanent reasoning given in the second part. It is this part that constitutes the original development and addition to the contemporary scientific debate particular to this project. This part first provides a description of different particular notions and formalisations in CTT

that are essential for the analysis of analogical argumentation. It then goes on by giving a general description of reasoning by analogy, both with characteristics and with heteronomous imperatives. This analysis provides us with a complex formula for analogical reasoning that will need a particular notational practice to be explained. The project continues by describing a way to translate this into a dialogical explanation. It then provides rules for eight different forms of analogical arguments. The permission of the analogical argument will also be described as more refined in the dialogical conception compared to the general CTT approach, as we would allow the permitted analogies to be attached to the relevant particular form of analogical argument. This last part terminates by a discussion of the philosophically relevant aspects of the previously introduced analysis and for the projects choice of a dialogical interpretation of constructive type theory.

Intention, motivation and goal

The role of the logician in a practical context

When attempting to analyse a concept from a domain as a logician we are given a seemingly conflicting role. On one side we should attempt to base the analysis on the actual practice, so that the analysis reflects the use of this concept. On the other side we are also dealing with a domain that seems to have some normative character. The analysis should not simply be an empirical investigation of how the concept is used, but it should also provide some guidelines for distinguishing good from bad practice of this concept. In this sense such project seems to be in conflict. Should our project only describe the actual practice and risk not being useful because of its lack of opinion regarding the practice? Or, should our project only describe the normative foundations for this concept and thereby risk to end up too far away from the concept we originally wanted to analyse?

These questions do not seem to be particular to logic, but rather as attached to something more general regarding the practice of doing philosophy. Philosophical concepts should ideally not be too far away from what we normally understand by these concepts, while at the same time they should make us able to explain a correct use. *Truth* is an obvious example of this problem. A proper philosophical definition of truth should in the same time be able to accommodate our intuitions about what truth is and give us guidelines for a correct usage of such concept. This might be reduced to a question about the relationship between normativity and reality, though it is a problem that one would need to overcome in order to provide the kind of analysis that is intended in this projected. Legal theory does not seem to be any exception.

The role of logic in legal theory seems indeed to be affected by these mentioned problems. Logic is on one hand expected to show the actual practice, while on the other hand also expected to have some effect on the practice. It is not sure that there is one specific way to solve this problem. The best one can do is, as a good legal practitioner, to show discretion. By showing discretion, one can hopefully arrive at an analysis that is neither too far away nor too close to actual practice so that our analysis will be based on reality, without losing the normativity that characterises philosophical and logical concepts.

Why immanent reasoning?

There are multiple reasons for choosing precisely immanent reasoning to represent analogies and a thorough explanation of these reasons is given in chapter 9.

The most important reason is the framework's particular way of explaining meaning. In most classical frameworks there is a clear distinction between the syntax and the semantics, or the form and the meaning. When an expression is assigned a meaning, it is assumed that this expressions is well-formed. CTT has a different approach to this. Here, the form of an expression is not simply

assumed to be made in a correct way, but the explanation of this form is also included in the object-language by means of formation rules. Together with the constructive aspect, this enables the framework to provide a clear distinction between the Aristotelean concepts of *meaning*, *actuality* and *potentiality*. CTT is then able to express a very sophisticated notion of conditional that includes dependent types, which will show to be essential for the present formalisations of juridical concepts. It is this notion of dependent types that enables the implementation of *initial conditions* for analogical reasoning.

The introduced analysis is based on a formalisation of a special kind of conditional, called *conditional right* or *moral conditional*. Leibniz analysed these conditionals by imposing some particular requirements that would distinguish them from other conditionals. CTT has proven to be a powerful tool for precisely capturing many important aspects of Leibniz's analysis. The present project utilises this CTT formalisation of moral conditionals as a foundation for its analysis of analogical reasoning. One result of this project is to show that by embedding the expression of one moral conditional inside another, we can achieve a formalisation of the procedure for analogical reasoning. An argument by analogy might therefore be said to be a special and complex form of moral conditional.

The third reason for utilising immanent reasoning is related to the meaning explanations provided by CTT, and particularly by its dialogical interpretation. By means of the hypothetical judgment, CTT enables us to capture not only corresponding truth conditions as for the classical material conditional, but also the precise *dependency* that the consequent has on its antecedent. This is what makes CTT so expressive regarding the formalisation of juridical and moral claims. We can then show how a decision is dependent on its reason and how the deontic qualification is dependent on the performance of the action in very precise ways. Its dialogical

interpretation provides a natural and comprehensible framework for meaning explanations that is closely linked to actual legal practice and that unites logical inferences and argumentation theory in one single framework.

Academic context

The project positions itself in a contemporary debate regarding the logical analysis of analogies and analogical reasoning. The goal of the project is twofold. It first intends to present and compare contemporary theories for reasoning by analogy. Second, it intends to provide an independent analysis of such reasoning based on the framework of immanent reasoning. This is based on the comparison of the different theories as it attempts to include important aspects introduced in the contemporary debate, while at the same time provide a more refined analysis accounted for by the particular dependency found in analogical reasoning.

The dialogical approach to logic and argumentation can be traced back to antiquity, by the works of Aristotle and Plato. Logic was then considered to be an activity that was performed as a dialogue regarding some proposition. In the modern approach introduced by Frege, the view on the role of logic changed to become a question about abstract manipulation of formulas. The contemporary dialogical approach was introduced by Lorenzen (1961) who brought back the antique idea to again consider logic as dialectical by utilising a game-theoretical approach to meaning. This modern dialogical approach has been further refined and developed into a framework where a great variety of logical systems have been interpreted and compared, creating multiple branches of dialogical logics. One of these branches is what is now called immanent reasoning. It is the result of describing an intimate connection between dialogues, constructivism and intuitionism, particularly related to the research done in the University of Lille. The goal of this project is to enter into this tradition and show how the framework can be applied also in the context of analogical reasoning in law.

The project introduces a new analysis of analogies in immanent reasoning. The particularities of this framework enables us to provide a formalisation of analogical reasoning in a precise way in line with corresponding contemporary analyses in other frameworks. Furthermore, this project utilises the concept of dependent types to introduce a condition of initial permission for the use of analogies, not introduced in any previously given logical analysis. This condition is indicated by different theorists, particularly from legal scholars, though from the logical perspective this seems to be the first time that this condition is included in the explicit logical representation. It enables the analysis to not only account for the use of analogy, but also to explain the introduction of the analogical argument in the first place.

Part III

Analysing analogy

Chapter 7

CTT analysis

We might distinguish two kinds of analogical argumentation or reasoning with precedents. We have reasoning about heteronomous imperatives, together with the more general reasoning about characteristics. The difference between the two is what kind of result we achieve from the analogical argument. In the case of reasoning about heteronomous imperatives, the result of the argument should be an imperative, understood as a decision whether the performance of a certain action is law-breaking or law-abiding. In reasoning about characteristics, the result is whether a certain situation has a given property. Typically, this is the situation for reasoning about definitions or borderline rules. In the intuitionistic framework, imperatives seem to be a special kind of predicate. Reasoning about heteronomous imperatives is therefore considered to be a special kind of reasoning about characteristics.

The requirement of efficiency consists of two conditions, the condition of co-extensiveness and the condition of co-exclusiveness. It is meant to provide restrictions on the choice of the occasioning characteristic. The occasioning characteristic should be so that in all cases where we have this occasioning characteristic, we also have the entailed characteristic, and in all cases where we do not have this occasioning characteristic, we do not have the entailed characteristic

either. Instead of being implemented explicitly, the condition of efficiency is included in the procedure as a whole. This analysis will be not be based on the requirement of efficiency as such, but rather a variant of the *Proportionality-principle*, namely:

Treat like cases alike and unlike cases differently.

This is a principle that can be traced back to Aristotle's *Nicomachean Ethics* (V.3,1131a10-b15) and provides a fundamental notion regarding equality, based on the notion of proportionality. It is often considered to be the foundation of analogical reasoning and reasoning with precedents, or sometimes even as a fundamental principle of law. It also provides justification for the principle of *stare decisis*. We can see that this principle actually consists of two parts, what we will call the *Alike-principle* and *Differently-principle*. The first can be described as:

Alike-principle: *Treat like cases alike.*

That a case is like another is understood as it being similar to the other. All cases are similar in one way or another and we cannot include all ways cases can be like each other. In order for this principle to be meaningful, we must therefore speak about *relevant similarities* or *similar relevancies*. A similarity is then based upon a relevant characteristic, shared by both cases and this characteristic should be the reason for its precise treatment.

We can easily see how this principle relates to analogical reasoning as an analogy is provided by two things that are similar in some aspect. If we have decided that a case is similar to another one, we should by the principle of equality, treat the first case in the same way as we treated the other. This provides the justification for arguments based upon similarity, namely positive analogies. However, we do also want to include arguments based upon differences, namely negative analogies. For this we would need to rely on the second part of the Proportionality-principle, which will be called

the *Differently-principle*,

Differently-principle: *Treat unlike cases differently.*

At first sight, this is a seemingly more controversial principle than the previous. However, we seem to have good reason also to accept this principle if we accept the first. Seemingly, no cases are the same; all cases are different to each other in some way or another. As with the first principle, we would therefore be inclined to speak about *relevant differences*, or *different relevancies*. That a case A is different from another case B is understood as A not sharing the relevant aspect with B that caused B being treated in the way it was. This means that there was some aspect in B that was the reason for this particular treatment and this aspect was not found in A. When we do not have the reason for this particular treatment, the treatment is seemingly groundless and therefore not applicable. That the treatment is not applicable means that we will have to find some other treatment, namely to treat it differently.

Together, these two principles provide justification for this analysis of analogical reasoning. The two principles seem to reflect the requirement of efficiency, though the Proportionality-principle is formulated closer to Aristotle's original analysis. By distinguishing the Proportionality-principle into the Alike-principle and the Differently-principle, we also provide the grounds for distinguishing between positive and negative analogies. This principle will be used to reflect the last step of the analysis performed here, namely the application to the target case.

7.1 General precedent-based reasoning

Performing analogical reasoning

The first step in the analysis will be to describe an informal seven-step procedure for how to perform general analogical reasoning with characteristics. The last step of the procedure might be said to be the most complex and controversial step as it involves the application to the target case. This step is a twofold step where the first part involves what we might call a standard or positive analogy (based on the Alike-principle), while the second part involves what we might call a negative analogy (based on the Differently-principle).

Procedure for performing analogical reasoning:

1. Include a target case where the presence or absence of some (consequent) characteristic has to be decided.
2. Find a relevant (occasioning) characteristic that will be chosen for reaching a decision in the target case.
3. Make sure that the terms are well-defined and that the use of the analogical argument is legally acceptable in this particular situation.
4. Decide whether this occasioning characteristic is present in the target case or not.
5. Find some source case and decide whether this occasioning characteristic is present in the source case or not.
6. Decide whether the entailed characteristic is present in the source case or not.

7. Decide whether the occasioning characteristic has the same status in both the target case and the source case (that it is present or absent). This is a twofold step:

    a) If the occasioning characteristic has the same status in both the source case and the target case (that it is present or absent):

        i. the status of the entailed characteristic in the source case can be transferred directly to the target case;

    b) If the occasioning characteristic has a different status in the source case and the target case (that it was present or absent):

        i. the status that the entailed characteristic has in the source case should not be the situation in the target case (its negation can be transferred to the target case).

Explaining the procedure

The first step is the foundation for the argument in the first place as there has to be some particular case that motivates the introduction of the argument. This is the target case, which essentially can be described as a problem that has to be solved.

The second step is the most difficult and controversial step when speaking about analogical reasoning. This step corresponds to what Brewer (1996) refers to as an abductive step, as it is the introduction of the occasioning characteristic. The choice of relevant occasioning characteristic seems to depend on some creative or intuitive aspect that cannot be fully described by a procedure. We might use constraints similar to ones developed for abductive inferences, like simplicity, generality, coherence and possibly particular constraints related to legal reasoning, but even with such constraints it seems difficult or impossible to describe an efficient procedure for choosing such characteristic. However, if we have found a potential characteristic, we might reject it if it does not give a coherent

result. This might motivate us to go back to change or revise the originally chosen characteristic. This is a result of the efficiency requirement that is implemented in the analysis by means of the Proportionality-principle.

The third step is the starting point for the analysis. The occasioning characteristic chosen in the second step has to be well-defined, which in CTT means to be well-typed. The legal result of the occasioning characteristic and its negation must be type-declared. We need to declare a set for the accepted source cases. We must also declare that the legal system permits the use of analogical reasoning in this particular situation. This can involve the proposed requirements mentioned earlier, but they can also be different.

The fourth step is the decision of whether the occasioning characteristic or its negation is present in the target case. This is the first *investigation* that is represented in the analysis.

The fifth step refers to the source case. The same investigation has to be performed in the chosen source case, whether the chosen occasioning characteristic or its negation is present in the source case.

The sixth step is a second investigation in the source case and creates the foundation for the decision in the target case. This investigation relates to whether the presence of the occasioning characteristic, or the absence of the occasioning characteristic, provides the presence or absence of the entailed characteristic in the source case.

The seventh step is the most complex step as it is the application of a consequence in the target case. It is a twofold step where the first part explains what should happen if the occasioning characteristic either is present in both the target case and the source case or is absent in both the target case and the source case. In this situation, we might directly transfer the presence/absence of the entailed characteristic from the source case to the target case. This is what might be called a positive analogy. It is a result of the Alike-principle, to treat like cases alike. The second alternative

describes a negative analogy where the occasioning characteristic is present in either the target case or in the source case and it is absent in the other. In this situation we might infer that the negation of the presence or absence of the entailed characteristic in the source case, holds in the target case. This is a result of the Differently-principle, to treat unlike cases differently. It is in the interpretation of this step that we notice a difference between intuitionistic and classical logic. Since in classical logic we have the elimination rule of double negation, we might infer that when the entailed characteristic is absent in the source case, it should be *present* in the target case. This is not the case in intuitionistic logic. Intuitionistically, we can only infer that the entailed characteristic should *not be absent*, and this is not the same as to say that it should be present.

Positive and negative analogies

Positive analogies occur when the status of the chosen occasioning characteristic is shared between the source and the target. We can then infer that the entailed characteristic has the same status in the target case as it had in the source case. If the entailed characteristic is present in the source case, we can infer that it should be present in the target case. While if the entailed characteristic is absent in the source case, we can infer that it should be absent also in the target case.

Negative analogies occur when the status of the chosen occasioning characteristic is different in the source and the target case. For negative analogies, when the entailed characteristic is present in the source case, we can infer that this characteristic should be absent (its negation should be present) in the target case. Similarly, if the negated entailed characteristic is present in the source case, we can infer that this negated characteristic should be absent (its negated should be present) in the target case. We assume that the absence of a characteristic is the same as the presence of the negated characteristic. That a characteristic is present is denoted as $B(x)$ while that a characteristic that is absent is denoted as $\neg B(x)$. In

negative analogies, when we have a presence, $B(x)$, of an entailed characteristic in the source case, we can infer that this characteristic should be absent, $\neg B(x)$, in the target case. A particular situation occurs when we have an absence, $\neg B(x)$, of an entailed characteristic in the source case. We can then infer that this (absent) characteristic should be absent, $\neg\neg B(x)$, in the target case. This is not the same as saying that the entailed characteristic should be present in the target case, as we then end up with a double negated characteristic in the target case. Since we are in an intuitionistic framework, we cannot infer the non-negated characteristic from this. This means that in negative analogies, we end up with a double negated characteristic in the target case when we have a negated characteristic in the source case.

The meaning of this double negated characteristic might seem slightly unclear at first sight. However, by a closer look this distinction seem to be rather natural in the legal context. Since the framework of immanent reasoning is constructive, the notion of truth is connected to its provability. That something is true means that it can be proven. That a negation is true means that the non-negated is not provable (that the attempt of proving is aborted). That a double negation is true means that the negated is not provable. In a legal context, this means that we have a refutation of the negation, but not an explicit proof for a (non-negated) decision. We are then provided with a distinction between reasons in favour of some claim and reasons against rejecting a claim. Negative analogies will generally provide reasons of the last kind.

Restricted and unrestricted analogies

The informal description describes how to reason when including one source case. This means that the result depends on the situation in a single source case, independently of what are the situations in all other source cases. Usually in legal reasoning, we would like to base the arguments on what generally holds, not only the situation of a particular source case. This can be included by restricting the analysis to include not a single source case, but all source cases.

Based on this, we can make a distinction between what we might call *restricted* and *unrestricted* analogical reasoning. The informal analysis of the procedure describes what happens in the unrestricted variant of analogical reasoning. This distinction can then be expressed in the following way:

Unrestricted analogy: An unrestricted analogy is an inference based on a single source case that share (or differ based on) a characteristic with the target case;

Restricted analogy: A restricted analogy is an inference based on all source cases that share (or differ based on) a characteristic with the target case.

The difference between the restricted and the unrestricted analogical reasoning is whether we require the analogy to hold for all cases or only for a single case, irrespective of all other cases. An unrestricted analogy is based on simply a similarity or difference between the source and the target. Since it does not limit the point of reference for the similarity or the difference, it is also vulnerable to the previously described argument of a potential infinite amount of similarities (or differences). This means that it can be used as an argument for any proposition, and that it therefore hardly can be used as a reference for the use of analogical reasoning in a legal context.

Since the unrestricted analogy only requires one source case, it is considered to be weaker than the restricted form. In ordinary life outside of the context of legal reasoning, it is the form that we often refer to when we speak about analogy. Because of this generality, an unrestricted analogical argument does not seem to provide very strong justification for its result. In short, the unrestricted analogy does not seem to include anything that corresponds to the requirement of efficiency.

A restricted analogy will provide a stronger justification and is also the form that is used in the legal context. Instead of being an analogy over a single source case, it quantifies over all source cases so that all source cases that are similar (or different) to the target case should be coherent in regard to the entailed characteristic of the analogy. However, it is important to note that also in a restricted analogy, the analogy depends on particular source cases and should not necessarily be considered dependent on the generalisation as such. This shows that there seem to be some tension inside the restricted analogy, whether we speak about a particular source case or a generalisation. The framework of immanent reasoning seems capable of capturing this in a rather subtle way as it allows the analogical argument to based upon a particular case, though captures the general aspect by the ability to *choose*. One could have chosen differently, but end up with the same result and after the choice is made, the analogy is dependent on the particular case that was chosen. In this way, immanent reasoning is able to capture both the particular and the general aspect of restricted analogical reasoning.

Imposing conditions on analogical reasoning

A widespread condition for analogical reasoning is the condition of efficiency. There is no explicit implementation of this condition in the process, but the condition of efficiency can be introduced by restricting step 5 to 7 to hold not only for one source case, but for all source cases, so that we speak about a *restricted* analogy. If

no decision can be reached based on all source cases, one has to go back to step 2, choose another characteristic and continue the process from there. Other conditions regarding the use of analogical reasoning can be implemented in step 3, as they will be analysed as formation conditions. The logical analysis provided in this work describes and analyses step 3 to step 7.

In the third step, the conditions regarding the formation of an analogy are implemented. This step could be said to consist of several substeps. The first is the type declaration of the proposition or characteristic. We represent the propositions and characteristics as the standard type *prop*. For some proposition A, we would need to suppose:

$$A : prop.$$

The source cases have to be declared as a set. It means that we have some defined and accepted source cases that might be used in the analogy.[1] This can be represented in the following way:

$$Source : set.$$

In addition, we have to declare that the intended entailed characteristic is a proposition. This proposition is also dependent on both the proposition A and on the set $Source$, introduced in the following way:

$$B(x, s) : prop(x : A, s : Source).$$

The last part is the inclusion of the permission in the legal system of utilising analogical reasoning in this particular situation. The permission of an analogy is rooted in its result in the target case, not in the proposition itself. This means that we will introduce

[1] In the CTT representation, we leave out the explicit representation of the target case as a set. This is to avoid further complexity and keep the analysis as simple as possible. In the dialogical analysis, complexity is however less of a problem and we will introduce the explicit set for the target case in this representation. See chapter 8 for details about the implementation of the target case set.

the permission of the analogy on the presence or absence of B. We therefore have four sets of permitted analogies, represented in the following way:

$$PA_1(z_1) : set(z_1 : (x_1 : A)B(x_1) \vee (x_2 : A)\neg B(x_2));$$
$$PA_2(z_2) : set(z_2 : (y_1 : \neg A)\neg B(y_1) \vee (y_2 : \neg A)\neg\neg B(y_2));$$
$$PA_3(z_3) : set(z_3 : (x_3 : A)\neg B(x_3) \vee (x_4 : A)\neg\neg B(x_4));$$
$$PA_4(z_4) : set(z_4 : (y_3 : \neg A)B(y_3) \vee (y_4 : \neg A)\neg B(y_4)).$$

The explicit permission of an analogy is introduced by the previously described judgments. These judgments can be produced by an instance of the elimination rule of the disjoint union. The instance for the first judgment is the following:

$$\frac{\begin{array}{ccc} & (x : B) & (y : \neg B) \\ c : B \vee \neg B & d(x) : PA(i(x)) & e(y) : PA(j(y)) \end{array}}{D(c, (x)d(x), (y)e(y)) : PA(c),} \; PA$$

which reads that when we have an a that is a permitted analogy, it is justified by the proof object $D(c, (x)d(x), (y)e(y))$ that is produced when c is the proof object of the disjunction and $(x)d(x)$ is verified in the case of B, while $(y)e(y)$ is verified in the case of $\neg B$.

It is important to note that this does not explain the content of this requirement, for example that it should be a lacuna in the law and not undermine constitutional values, but is rather a representation of the result assessing this content.

For general analogies, we then end up with the following context:

$$A : prop,$$
$$Source : set,$$
$$B(x, s) : prop(x : A, s : Source),$$
$$PA_1(z_1) : set(z_1 : (x_1 : A)B(x_1) \vee (x_2 : A)\neg B(x_2)),$$
$$PA_2(z_2) : set(z_2 : (y_1 : \neg A)\neg B(y_1) \vee (y_2 : \neg A)\neg\neg B(y_2)),$$
$$PA_3(z_3) : set(z_3 : (x_3 : A)\neg B(x_3) \vee (x_4 : A)\neg\neg B(x_4)),$$
$$PA_4(z_4) : set(z_4 : (y_3 : \neg A)B(y_3) \vee (y_4 : \neg A)\neg B(y_4)).$$

In the following sections, the explicit formulation of this context will be left out for the sake of simplicity.

Representing source cases

We can now describe how to reach a decision based on the available source cases at hand. Here, we suppose that the absence of a characteristic is the presence of its negation. That A is being absent will therefore be described as $\neg A$ being present. For a source case s, we then have several situations:

1. A is present in s,
   a) B is present in s,
   b) $\neg B$ is present in s;
2. $\neg A$ is present in s,
   a) B is present in s,
   b) $\neg B$ is present in s.

Depending on the source case s, we can use s as an argument for a certain standpoint in the target case. The standard form of analogical reasoning is based on A being present in both the source case and the target case and since B was present/absent in the source case, it should also be present/absent in the target case. If we assume that A is present in the target case, this can be represented in the following way, so that when there are two lines from a statement, it represents an implication of a conjunction:

For a source case s
- A is present in s
  - B is present in s
    - B should be present in the target case
  - $\neg B$ is present in s
    - $\neg B$ should be present in the target case.

In addition to the standard form of analogical arguments, we also have what we called negative analogies. Instead of being based on the similarity between the target case and the source case, it depends on their difference. If A is present in the target case and not in the source case, and B is present in the source case, B should be absent in the target case. And if B is absent in the source case, B should not be absent in the target case. We can represent this in the following way, where we still suppose that A is present in the target case:

For a source case s
- $\neg A$ is present in s
  - B is present in s
    - $\neg B$ should be present in the target case
  - $\neg B$ is present in s
    - $\neg\neg B$ should be present in the target case.

By combining these two representations, we end up with a procedure for handling analogical reasoning when A is present in the target case. This yields the following form:

For a source case s
- A is present in s
  - B is present in s
    - B should be present in the target case
  - $\neg B$ is present in s
    - $\neg B$ should be present in the target case
- $\neg A$ is present in s
  - B is present in s
    - $\neg B$ should be present in the target case
  - $\neg B$ is present in s
    - $\neg\neg B$ should be present in the target case.

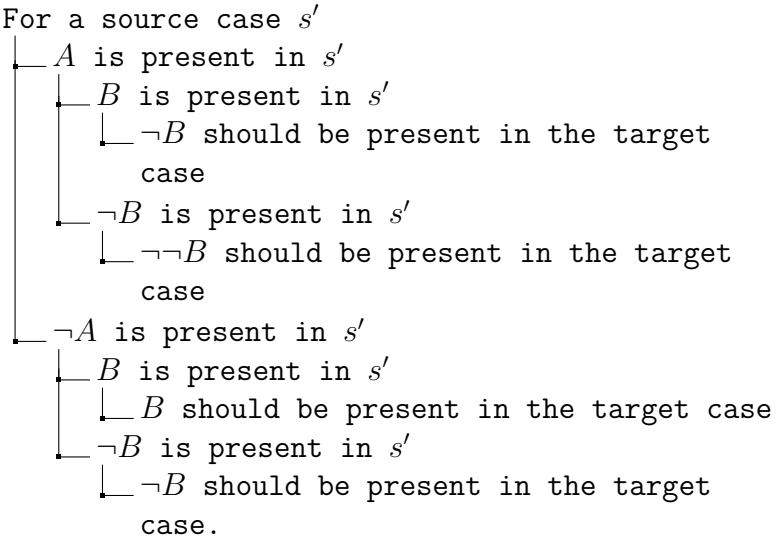
Correspondingly, we can describe the process of analogical reasoning when $\neg A$ is present in the target case. If $\neg A$ is also present in the source case, we speak about a positive analogy since $\neg A$ is shared between the source and the target. The decision whether B is present or absent can be directly transferred from the source case to the target case. We suppose here that $\neg A$ is present in the target case. This transfer can be represented in the following way:

For a source case s'
- $\neg A$ is present in s'
  - B is present in s'
    - B should be present in the target case
  - $\neg B$ is present in s'
    - $\neg B$ should be present in the target case.

When $\neg A$ is present in the target case, we might also speak about negative analogies. The negative analogy occurs when A is present in the source case. The analogy is then based on some characteristic that is not shared between the source and the target. This can be represented in the following way, where it is supposed that $\neg A$ is present in the target case:

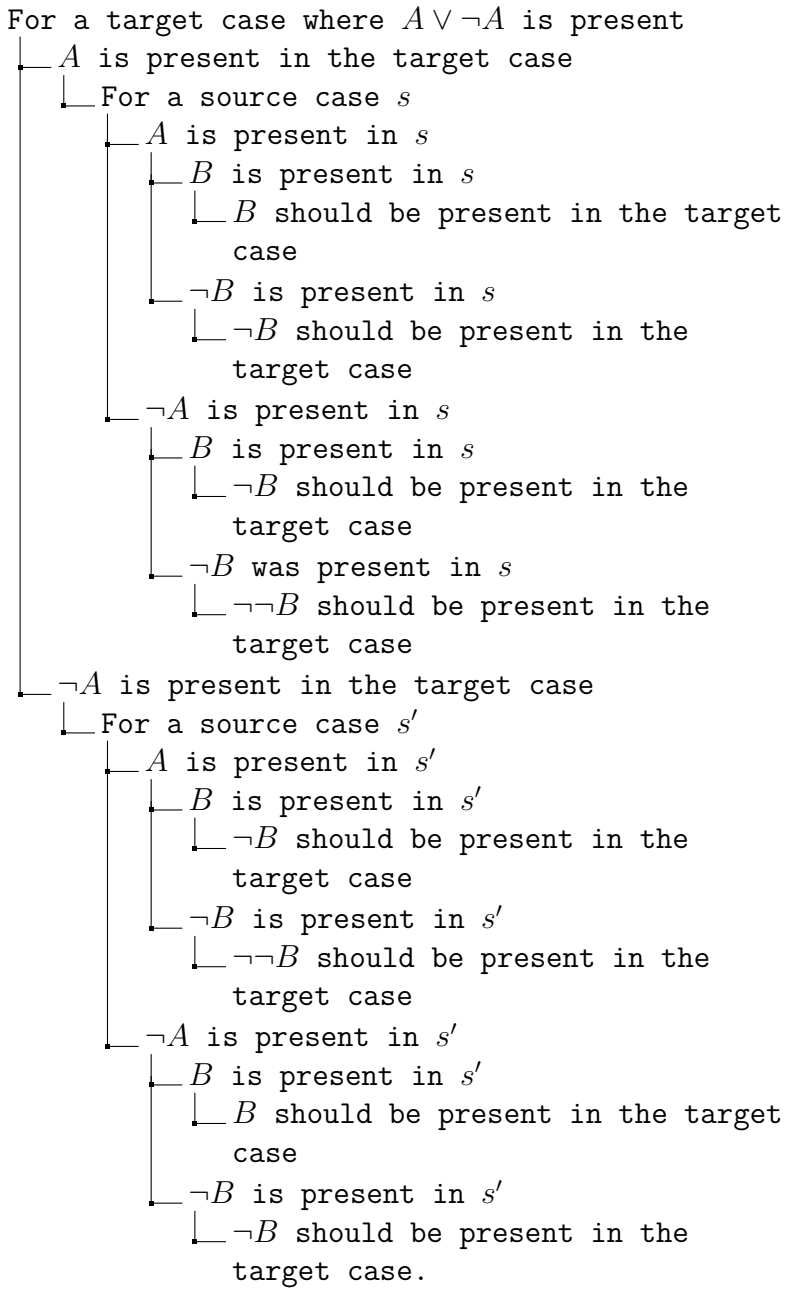
For a source case s'
- A is present in s'
  - B is present in s'
    - $\neg B$ should be present in the target case
  - $\neg B$ is present in s'
    - $\neg\neg B$ should be present in the target case.

We then end up with a procedure for handling analogical reasoning when $\neg A$ is present in the target case. This yields the following form:



Representing the analogical procedure in CTT

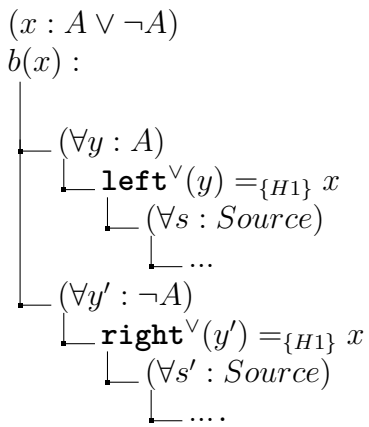
By combining the procedures for handling analogies when A is present and when $\neg A$ is present in the target case, we end up with a description of the whole process for analogical reasoning. This can be represented by the following:


For a target case where $A \vee \neg A$ is present
A is present in the target case
For a source case s
A is present in s
B is present in s
B should be present in the target case
$\neg B$ is present in s
$\neg B$ should be present in the target case
$\neg A$ is present in s
B is present in s
$\neg B$ should be present in the target case
$\neg B$ was present in s
$\neg\neg B$ should be present in the target case
$\neg A$ is present in the target case
For a source case s'
A is present in s'
B is present in s'
$\neg B$ should be present in the target case
$\neg B$ is present in s'
$\neg\neg B$ should be present in the target case
$\neg A$ is present in s'
B is present in s'
B should be present in the target case
$\neg B$ is present in s'
$\neg B$ should be present in the target case.

This provides the foundation for its representation in CTT. The target case can be represented by a similar form as the conditional analysis described in Rahman and Granström (2019), though including an explicit dependency on the source case. In a similar way as previously described, we will use $\{H1\}, \{H2\}, \ldots$ as abbreviations for formulas like $A \vee \neg A$ in identity statements. We then use a conditional formulation to represent the inquiry of whether it is the right or the left side of this disjunction that makes it true in the target case. If it is the left side A, we continue with the source case s. If it is the right side $\neg A$, we continue with the source case s'. This yields the following, incomplete formula:

$$b(x) : [(\forall y : A)\mathbf{left}^{\vee}(y) =_{\{H1\}} x \supset (\forall s : Source)...] \wedge$$
$$[(\forall y' : \neg A)\mathbf{right}^{\vee}(y') =_{\{H1\}} x \supset (\forall s' : Source)...](x : A \vee \neg A).$$

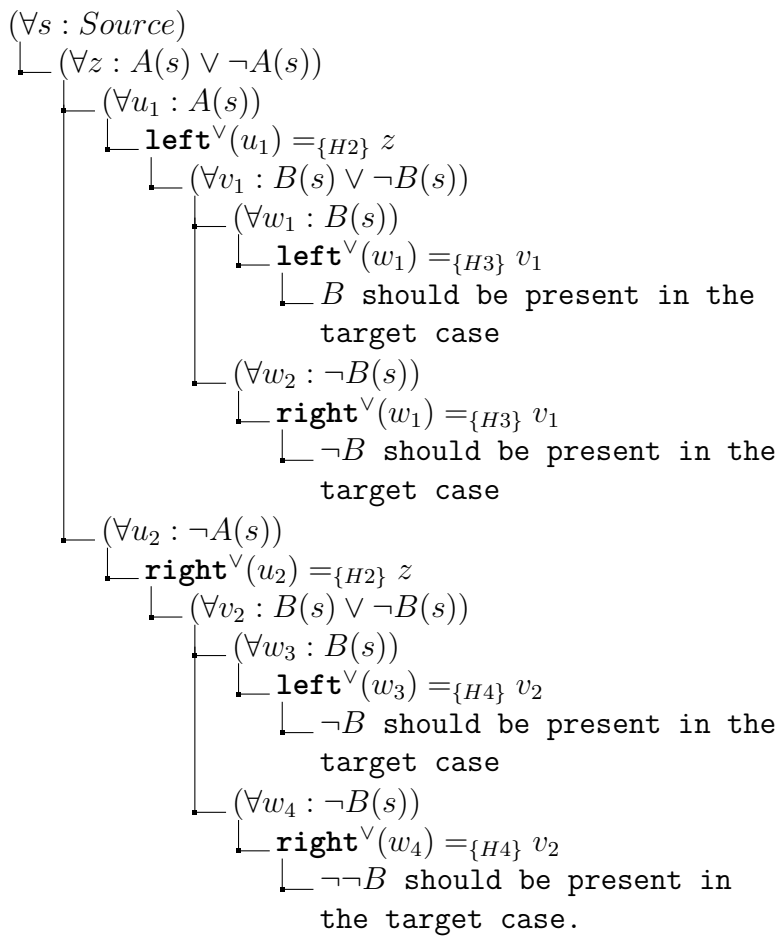
In the tree structure, this formula receives the following notation:



This formula is incomplete because it does not include the description of the source cases. They should be included in '...'. By understanding the target case in this way, we include a notion of *suspense*. Based on the procedure for performing analogical reasoning as described earlier, this representation will include a suspense on the decision of whether A or $\neg A$ is present in the target case. Essentially, this means that the presence or absence of A is not

given by the target case, but is performed as an investigation into the target case when referring to an analogy. We might therefore say that the investigation or inquiry whether A or $\neg A$ is present in the target case comes after the target case itself. The source cases are implemented inside the formula after it has been decided whether A or $\neg A$ is present in the target case. The source cases are only introduced when trying to build an analogical argument after deciding the presence or absence of some proposition in the target case. The analysis therefore does not introduce the source cases before they are needed.

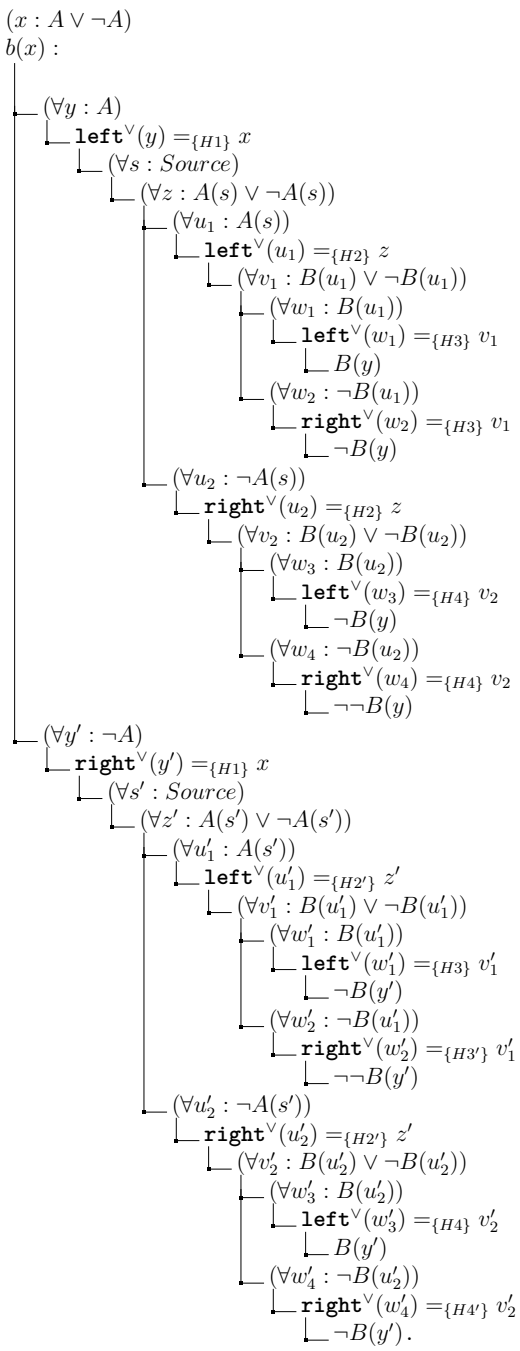
There are two '...' in the incomplete formula. The source cases are introduced in two parts, first when A is present in the target case and second when $\neg A$ is present in the target case. If A is present in the target case, the process can be formalised in the following way:



Similarly, if $\neg A$ is present in the target case, the process can be formalised in the following way:

- $(\forall s' : Source)$
  - $(\forall z' : A(s') \vee \neg A(s'))$
    - $(\forall u'_1 : A(s'))$
      - $\mathbf{left}^{\vee}(u'_1) =_{\{H2'\}} z'$
        - $(\forall v'_1 : B(s') \vee \neg B(s'))$
          - $(\forall w'_1 : B(s'))$
            - $\mathbf{left}^{\vee}(w'_1) =_{\{H3\}} v'_1$
              - $\neg B$ `should be present in the target case`
          - $(\forall w'_2 : \neg B(s'))$
            - $\mathbf{right}^{\vee}(w'_1) =_{\{H3'\}} v'_1$
              - $\neg\neg B$ `should be present in the target case`
    - $(\forall u'_2 : \neg A(s'))$
      - $\mathbf{right}^{\vee}(u'_2) =_{\{H2'\}} z'$
        - $(\forall v'_2 : B(s') \vee \neg B(s'))$
          - $(\forall w'_3 : B(s'))$
            - $\mathbf{left}^{\vee}(w'_3) =_{\{H4\}} v'_2$
              - B `should be present in the target case`
          - $(\forall w'_4 : \neg B(s'))$
            - $\mathbf{right}^{\vee}(w'_4) =_{\{H4\}} v'_2$
              - $\neg B$ `should be present in the target case.`

By combining both of these formulations, we end up with a CTT analysis of the process of analogical reasoning. The general formulation for analogical reasoning in CTT gives the following:


$(x : A \vee \neg A)$
$b(x)$:
$(\forall y : A)$
$\mathtt{left}^{\vee}(y) =_{\{H1\}} x$
$(\forall s : Source)$
$(\forall z : A(s) \vee \neg A(s))$
$(\forall u_1 : A(s))$
$\mathtt{left}^{\vee}(u_1) =_{\{H2\}} z$
$(\forall v_1 : B(u_1) \vee \neg B(u_1))$
$(\forall w_1 : B(u_1))$
$\mathtt{left}^{\vee}(w_1) =_{\{H3\}} v_1$
$B(y)$
$(\forall w_2 : \neg B(u_1))$
$\mathtt{right}^{\vee}(w_2) =_{\{H3\}} v_1$
$\neg B(y)$
$(\forall u_2 : \neg A(s))$
$\mathtt{right}^{\vee}(u_2) =_{\{H2\}} z$
$(\forall v_2 : B(u_2) \vee \neg B(u_2))$
$(\forall w_3 : B(u_2))$
$\mathtt{left}^{\vee}(w_3) =_{\{H4\}} v_2$
$\neg B(y)$
$(\forall w_4 : \neg B(u_2))$
$\mathtt{right}^{\vee}(w_4) =_{\{H4\}} v_2$
$\neg\neg B(y)$
$(\forall y' : \neg A)$
$\mathtt{right}^{\vee}(y') =_{\{H1\}} x$
$(\forall s' : Source)$
$(\forall z' : A(s') \vee \neg A(s'))$
$(\forall u'_1 : A(s'))$
$\mathtt{left}^{\vee}(u'_1) =_{\{H2'\}} z'$
$(\forall v'_1 : B(u'_1) \vee \neg B(u'_1))$
$(\forall w'_1 : B(u'_1))$
$\mathtt{left}^{\vee}(w'_1) =_{\{H3\}} v'_1$
$\neg B(y')$
$(\forall w'_2 : \neg B(u'_1))$
$\mathtt{right}^{\vee}(w'_2) =_{\{H3'\}} v'_1$
$\neg\neg B(y')$
$(\forall u'_2 : \neg A(s'))$
$\mathtt{right}^{\vee}(u'_2) =_{\{H2'\}} z'$
$(\forall v'_2 : B(u'_2) \vee \neg B(u'_2))$
$(\forall w'_3 : B(u'_2))$
$\mathtt{left}^{\vee}(w'_3) =_{\{H4\}} v'_2$
$B(y')$
$(\forall w'_4 : \neg B(u'_2))$
$\mathtt{right}^{\vee}(w'_4) =_{\{H4'\}} v'_2$
$\neg B(y')$.

This procedure is the CTT representation of step 4 to 7 in the description of the process of analogical reasoning. By including the context that was described earlier, we would also include step 3. When applying this formula to some particular case, every disjunction will play the role as a question, inquiry or investigation. After step 3, the first investigation that occurs is formalised as $x : A \vee \neg A$ and represents the decision whether the chosen characteristic is present or absent in the target case. After it has been decided whether A is present or absent in the target case, the source cases are introduced and for every source case the investigation whether the characteristic is present or absent in this source case is formalised $(\forall z : A(s) \vee \neg A(s))$ and $(\forall z' : A(s') \vee \neg A(s'))$. After deciding whether A is present or absent in the source case, the next investigation is whether the entailed characteristic is present or absent in the source case, which is represented as $(\forall v_1 : B(u_1) \vee \neg B(u_1))$, $(\forall v_2 : B(u_2) \vee \neg B(u_2))$, $(\forall v_1' : B(u_1') \vee \neg B(u_1'))$ and $(\forall v_2' : B(u_2') \vee \neg B(u_2'))$. If the occasioning characteristic is present in both the target case and the source case, the presence (or absence) of the entailed characteristic in the source case can be directly transferred to the target case. This is the situation for w_1, w_2, w_3' and w_4'. This transfer is formalised by binding the entailed characteristic to the choice that was performed in the target case, y or y'. This corresponds to what we have called a positive analogy.

The other alternative is that the characteristic is present in either the target case or the source case and not present in the other. This is the foundation for what we have called negative analogies. Negative analogies are however slightly more complicate than positive analogies. The negative analogies occur in w_3, w_4, w_1' and w_2'. In w_3, we have $\neg A$ and B in the source case, so since we also have A in the target case, we can infer $\neg B$ in the target case. In w_4, we have $\neg A$ and $\neg B$ in the source case, so since we also have A in the target case, we can infer $\neg\neg B$ in the target case. In w_1',

we have A and B in the source case, so since we also have $\neg A$ in the target case, we can infer $\neg B$ in the target case. In w_2', we have A and $\neg B$ in the source case, so since we also have A in the target case, we can infer $\neg\neg B$ in the target case.

We notice that since we operate within an intuitionistic framework, the notion $\neg\neg B$ is not equivalent or reducible to B. We do however claim that this distinction is important as it highlights the kind of evidence provided by such negative analogies. A source case, where the point of reference is not shared with the target case cannot be used to provide evidence for applying a notion not present in the source case to the target case. It seems all in all to correspond well together with legal practice regarding evidence as it highlights the difference of being evidence for a certain claim and being evidence against its negation.

7.2 Precedent-based reasoning with imperatives

Performing analogical reasoning

Procedure for performing analogical reasoning

A more specific variant of analogical reasoning is reasoning about heteronomous imperatives. What is understood by imperatives is discussed in section 6.1. When performing reasoning with precedents about imperatives we seem to be entitled to some further inferences compared to the situation with characteristics. In this representation, we will base it on the description on the three deontic categories described earlier. If we had included all five categories by Ibn Ḥazm, we would indeed arrive at a different process as there would be a distinction between an action being law-abiding and an action being legally neutral.[2] Note also that we here speak about *actions* rather

[2]For an analysis in the same system based on these five categories, see Kvernenes (2021).

$$LB(x) = \neg LA(x) : prop[x : A].$$

The first reads that A not being law-breaking is equal to A being law-abiding and the second that A being law-breaking is equal to A not being law-abiding. Generally, this seems to hold as it lies in the definition of both law-abidingness that it is not law-breaking and in law-breakingness that it is not law-abiding. They are incompatible concepts by definition. By including this definitional equality, we are able to explain the special inferences that analogies with imperatives seem to enable.

We can then treat the deontic qualifications as any other proposition when doing the analogical analysis. If the result in the target case is either that the performance or non-performance of A is not law-breaking, we can transform this result into it being law-abiding in the target case. Similarly, if the result in the target case is either that the performance or non-performance of A is law-abiding, we can transform this result into it not being law-breaking. The first might typically be used to establish the law-abidingness of A while the second might be used to establish the law-breakingness.

8.3 Dialogical example

Steamboat example

We will illustrate this approach by the steamboat example, *Adams v. New Jersey Steamboat Co.*, 151 N.Y. 163 (1896). The example is previously introduced and analysed in section 7.3. The example is a case about whether or not a certain steamboat owner is liable for a theft from a customer.

Following the previous treatment of this example, $F : prop$ stands for the proposition *refusing strict liability for the theft of valuables when the client paid for a room for some specified reasons and that the company has tempting opportunity for fraud and plunder of the client*. In the dialogical approach, we will let i represent the innkeeper case where the presence of F was law-breaking and r represent the

railway case where the absence of F was law-abiding. We can here recognise this argumentative process as a case of the Proponent suggesting a PPP-analogy between F and law-breakingness, based on i, and where the Opponent attempts to utilise r as an unsuccessful counterexample by a PAA-analogy.

Formation and permissibility

The first step is to establish the formation and general permissibility of the use of the analogical argument. This means to ensure that all terms are well-typed and that the use of the particular form of analogy is legally permissible in this precise context. This is analysed in Play 10.

We can see that this play closely resembles the example in Play 9. The only difference is that we in this example discuss a particular characteristic F and its law-breakingness, rather than the characteristic B that is dependent on the characteristic A. Similar explanations of the different moves will therefore also hold here. We will simply focus the attention on the most important aspect of this constructive approach, namely the permission of the analogy. The permission of the PPP-analogy is here introduced in the initial concessions. The permission of the PAA-analogy is instead introduced by the Opponent when proposing this form as a counterargument.

In the initial concessions, we only introduce the particular permission that is needed for this particular argument, as other forms of analogy could (at least conceptually) not be permitted. There might be a discussion whether this actually can be reflected in the legal system, though we seem to safely claim that the permission of other forms of analogy might not have been *assessed*. We could have chosen to also include this second permission in the initial concessions, but by introducing it in the course of the play, our approach seems more closely connected to actual legal practice where it is the proposer of the argument that carries the burden to show that this argument is legally permitted.

O			**P**		
I	$F : prop$				
II	$Tar : set$				
III	$Sou : set$				
IV	$LB(x, y) : prop[$ $x : F \vee \neg F, y : Tar \vee Sou]$				
V	$PA_1(z) : set[$ $z : (x : F)LB(x)$ $\vee\, (y : F)\neg LB(y)]$				
				! Analogy$[F, LB]$	0
1	n:=4			n:=5	2
3	$?_{prop}$	0		Analogy$[F, LB] : prop$	4
5	? $F_{Analogy[F,LB]1}$	4		$F : prop$	6
7	? $F_{Analogy[F,LB]2}$	4		$Tar : set$	8
9	? $F_{Analogy[F,LB]3}$	4		$Sou : set$	10
11	? $F_{Analogy[F,LB]4}$	4		$LB(x, y) : prop[$ $x : F \vee \neg F, y : Tar \vee Sou]$	12
13	? AnForm$[F, LB]$	0		! PPP-Analogy$[F, LB]$	14
15	$?_{prop}$	14		PPP-Analogy$[F, LB] : prop$	16
17	? $F_{PPP-Analogy[F,LB]}$	16		$PA_1(z) : set[$ $z : (x : F)LB(x)$ $\vee\, (y : F)\neg LB(y)]$	18
[Play develops normally]					
19	! PAA-Analogy$[F, LB]$	0			
21	PAA-Analogy$[F, LB] : prop$		19	$?_{prop}$	20
23	$PA_3(z) : set[$ $z : (x : F)\neg LB(x)$ $\vee\, (y : F)\neg\neg LB(x)]$		21	? $F_{PAA-Analogy[F,LB]}$	22
[Play develops normally]					

Play 10: Steamboat example, formation play

Justifying the analogy

In the formation play, no justification for the analogy is established. This is done in the particle play. This justificatory process has the form of the Proponent suggesting an analogy between F and being law-breaking, on the basis of a PPP-analogy. The Opponent tries

to provide a counterexample to this by a PAA-analogy, though fails since the PAA-analogy will not provide sufficient results to actually reject the PPP-analogy. The justificatory process is described in Play 11.

O			**P**		
I	$f : F \vee \neg F$				
II	$t : Tar(L^{\vee}(f))$				
III	$i : Sou(L^{\vee}(f))$				
IV	$d : LB(i)$				
V	$r : Sou(R^{\vee}(f))$				
				! Analogy$[F, LB]$	0
1	n:=2			n:=3	2
3	? AnForm$[F, LB]$	0		! PPP-Analogy$[F, LB]$	4
5	! $(\forall y : \{x : F \vert Sou(x)\})$ $LB(y) \supset$ $(\forall z : \{x : F \vert Tar(x)\})LB(z)$	4		! $LB(p_2^t)$	12
9	$p_1^t : (\forall z : \{x : F \vert Tar(x)\})$ $LB(z)$		5	$p_1^s : (\forall y : \{x : F \vert Sou(x)\})$ $LB(y)$	6
7	$i : \{x : F \vert Sou(x)\}$	6		$d : LB(i)$	8
11	$p_3^t : LB(p_2^t)$		9	$p_2^t : \{x : F \vert Tar(x)\}$	10
13	! PAA-Analogy$[F, LB]$	0			
[Opening of subplay]					
			13	! $(\forall y : \{x : \neg F \vert Sou(x)\})$ $\neg LB(y) \supset (\forall z :$ $\{x : F \vert Tar(x)\})\neg\neg LB(z)$	14
15	$p_1^c : (\forall y : \{$ $x : \neg F \vert Sou(x)\})\neg LB(y)$	14		$p_2^c : (\forall z : \{$ $x : \neg F \vert Tar(x)\})\neg\neg LB(z)$	16
17	$p_2^t : \{x : F \vert Tar(x)\}$	16		$p_4^t : \neg\neg LB(p_2^t)$	18
19	$p_5^t : \neg LB(p_2^t)$	18		$you_{gave\ up}(21) : \bot$	22
21	$p_6^t : \bot$		19	$p_3^t : LB(p_2^t)$	20

Play 11: Steamboat example

Describing the justificatory process

- I formulates the disjunction of the occasioning proposition and its negation. II states that F, the left side in I, holds in the steamboat case t. III states that the innkeeper case i is a source case and that F is present in this source case. IV states that F was law-breaking in i. V states that the railway case r is a source case and that F is absent in this source case.

- Move 0 states the proposed analogy, that there is an analogical relation between F and law-breakingness.

- In moves 1 and 2, the players choose their repetition ranks.

- In move 3, **O** challenges the proposed analogy in move 0 by the Analogy Challenge Rule 1. The Opponent here asks the Proponent to choose one analogy form that he thinks can be defended.

- In move 4, **P** chooses the analogy form to be a PPP-Analogy, where F is present in both the source case and the target case and where F was law-breaking in the source case. In terms of meaning explanations, it is here that the initial concessions III and IV are introduced.

- In move 5, **O** challenges **P**'s chosen analogy form in move 4 by the PPP-analogy Explanation Rule.

- Moves 6 to 11 show a normal development according to the rules of immanent reasoning. In move 6, **P** challenges the statement in move 5 by stating that the law-breakingness in the source cases is dependent on the presence of F. **O** challenges this statement in move 7 by stating the source case where F is present, here the innkeeper case i. In move 8, **P** defends this challenge by stating the law-breakingness in the source case from IV. In move 9, **O** defends the challenge in move 6 by stating that the law-breakingness in the target

case depends on the presence of F. Move 10 establishes the presence of F in the target case by **P** challenging the statement in move 9. In move 11, **O** defends the challenge in move 10 by stating the law-breakingness in the target case.

- In move 12, **P** then defends the challenge in move 5 by stating the law-breakingness in the target case. **P** can do this because of **O**'s statement in move 11. This finishes the first analogy form.

- In move 13, **O** challenges the thesis in move 0 again. This can be done because of **O**'s choice of repetition rank in move 1. The challenge is by the Analogy Challenge Rule 2, which now can be used since **O** already challenged by the Analogy Challenge Rule 1. This enables **O** to bring up a counterexample to the proposed analogy. **O** chooses to bring in the PAA-analogy form as a proposed counterexample. This move opens up a subplay where the proposed counterexample will be brought in. It is here that the initial concession V is introduced.

- In move 14, **P** challenges the proposed analogy form in move 13, by the PAA-Analogy Explanation Rule.

- Moves 15 to 22 follow normal development according the rules of immanent reasoning. In move 15, **O** challenges the statement in move 16, claiming that an absence of law-breakingness is dependent on the absence of F in the source cases. **P** defends this challenge by stating that the double negated law-breakingness is dependent on the absence of F in the target case. **O** challenges this statement in move 17 by stating the presence F in the target case. In move 18, **P** defends the challenge in move 17 by stating the double negated law-breakingness in the target case. In move 19, **O** challenges the statement in move 18 by stating that it is not law-breaking in the target case. In move 20, **P** states that the target case

is law-breaking. **P** can do this because **O** already stated this in move 11. In move 21, **O** is forced to posit $\perp$. This gives **P** the possibility to state the special play object $you_{gave\ up}(21)$ and the play terminates.

- **P** has the last word in the play and **O**'s proposed counterexample is therefore not effective and **P** wins the play.

We can see that this corresponds well together with the intended result, where the steamboat case was decided to be similar to the innkeeper case and not to the railway case.

Mortgage loan example

This section describes and analyses a particular example taken from Spanish Law, regarding Supreme Court decisions on the duty of paying the IAJD (Impuesto sobre Actos Jurídicos Documentados [Tax on Documented Legal Acts] related to the loan. The precise analysis of this particular case requires moves that go beyond the moves that are presented here and we will therefore provide only a semi-formal presentation of this example. The goal of this example is to show the role the different moves have in a concrete example taken from a contemporary civil law system.[6]

The cases

Actually, there are three main cases. However, all of them can be conceived as different plays on deciding about the interpretation of the Law concerning who must pay some particular tax specific to loans linked to a mortgage (either a mortgage loan or a credit

[6]The analysis and the description in this section is a modified analysis taken from «Elements for a Dialogical Approach on Parallel Reasoning. A Case Study of Spanish Civil Law» by Martínez-Cazalla, Menéndez-Martín, Rahman, and Kvernenes (2022).

Conclusion

Project

The present project is twofold. First, it gives a thorough representation of the concept of analogical reasoning in law by introducing and comparing contemporary theories. Second, it provides an independent analysis of analogical reasoning in the framework of immanent reasoning.

The first part explained and compared six contemporary theories of analogical reasoning in law. These theories were categorised into *schema-based* theories and *inference-based* theories. Schema-based theories of analogical reasoning capture the notion of analogy by a description of a rule or schema. Inference-based theories on the other hand explain analogical reasoning as a distinct way of reasoning. We identified the theories by Brewer (1996), Alchourrón (1991) and Woods (2015) to be schema-based and the theories of Bartha (2010), Prakken and Sartor (1996) and Rahman and Iqbal (2018) to be inference-based. We then compared the theories by how they handle the notions of *horizontal* and *vertical* relations and by how they analyse *multiple, competing* analogies.

The second part of the project was to provide an independent analysis of reasoning by analogy by utilising the framework of immanent reasoning. Immanent reasoning was described, together with an informal explanation of the other relevant notions of *case*, *relations* and *initial conditions*. The project then introduced two kinds of analogical reasoning, *general precedent-based reasoning* and

precedent-based reasoning with heteronomous imperatives. These kinds were first analysed in the general formulation of constructive type theory and then given an alternative formulation in the dialogical interpretation. Following this, we introduced a discussion on the advantages of utilising immanent reasoning as a framework for analysing legal reasoning in general and analogical reasoning in particular.

Results

The first goal is to describe analogical reasoning by introducing and comparing different contemporary theories of analogical reasoning in law. The second goal is to provide an independent analysis of analogies.

Even though the different contemporary theories had greatly different starting points, they all provided thorough and deep analyses of the concept of analogy. The schema-based theories provided explanations of both the horizontal and the vertical relations as either explicit or implicit *formal structures*. The inference-based theories on the other hand reduced the question of the horizontal relations to be a question of *identity* or *similarity*. The vertical relations were then described by a particular form of logical *dependency*. Across the categorisations of schema-based and inference-based theories, the theories differed in how they handled multiple, competing analogies. Two leave this notion *unexplained*, two introduce a particular *higher-order* operator on the different analogies and two consider multiple competing analogies as something that should motivate a *change* of the original analogy.

Based on a variant of the principle of *proportionality*, the present project provided an analysis of analogy in the framework of immanent reasoning. By utilising the formalisation of moral conditionals where one formulation is embedded in another formulation, we showed how we could represent analogical reasoning. Because of the particular notion of dependent types in CTT, this approach

also allowed for formalising *initial conditions* by an explicit notion of *permitted analogies*. This is a new feature, not previously known to have been introduced in any contemporary analysis of analogy. The dialogical interpretation takes this one step further, as this does allow for representing this feature as an individual condition for the particular *form* of the introduced analogy. This was done by distinguishing in total eight different forms of analogies. The dialogical interpretation also enabled the unification of general precedent-based reasoning and precedent-based reasoning with heteronomous imperatives in a simple way. We have shown that the framework of immanent reasoning is a powerful tool to handle analogical reasoning, which also seem capable of analysing inferences in law more generally.

Further research

In the contemporary legal discourse, a notion that is often considered to be closely connected to reasoning by analogy is *balance of interests*. This is an aspect that is often given considerable practical attention when solving a legal issue, a point highlighted by Armgardt (2022). Here, we have not attempted to include interests in the analysis. Though because of its close relationship with interpretation of precedents and analogical reasoning, it would indeed seem to be an aspect worth considering in an extended analysis of legal reasoning.

One of the particularities of the present analysis is its ability to express initial conditions in the formalisation. However, the precise *content* of these conditions stays to a large extent unexplained. A natural continuation of this project would then be to analyse the exact content of the initial conditions. This could then show the effects the no-answer question and the requirement of no-constitutional restraints could potentially have on the analysis.

Furthermore, the present project briefly described the notion of *precedent*. It identified the conditional structure found in legal cases, though a precise analysis of precedents and cases would seem valuable not only for the understanding of analogies in law, but generally for all kinds of legal reasoning. Precedents and legal cases provide a significant aspect of most contemporary legal systems and a thorough analysis of such notions in CTT could provide a deeper understanding of the logical interactions that take place between reasoning, logic and law.

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